ijfcar.azurewebsites.net
International Journal for Computer Application and Research
Vol. Special Issue 1, February 2013 (ISSN: 2320 –5067)

Pairwise g**-comapct modulo *I*, countably compact modulo *I*, g**-Lindeloff modulo *I* Spaces

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Abstract: In this paper, pairwise g**-comapct modulo *I* space, pairwise g**-countably comapct modulo *I* space, pairwise g**-Lindeloff modulo *I* space are introduced and the relationship between these concepts are studied.

Keywords: *Pairwise g**-comapct modulo I space, pairwise g**-countably comapct modulo I space, pairwise g**-Lindeloff modulo I space.*

1. INTRODUCTION

Levine **[3]** introduced the class of g-closed sets in 1970 and M.K.R.S. Veerakumar**[11]** introduced g*-closed sets in 1991. Ideal topological spaces have been first introduced by K. Kuratowski **[4]** in 1930. We have introduced and studied g**-closed sets **[5]**,g**-compact modulo *I* spaces and g**-countably compact modulo *I* spaces**[6]**, g**-Lindeloff spaces**[8]** and g**-Lindeloff modulo *I* spaces**[9]**. In this paper pairwise g**-comapct spaces, pairwise g**-countably compact spaces, pairwise g**-Lindeloff spaces, pairwise g**-comapct modulo *I* spaces, pairwise g**-countably compact modulo *I* spaces, pairwise g**-Lindeloff modulo *I* spaces are defined and their properties are investigated.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space(X, τ) is called

1) *generalized closed* (briefly *g-closed*)**[3]** if cl(A) □ U whenever A □ U and U is open in (X, τ).
2) *generalized star closed* (briefly *g*-closed*)**[11]** if cl(A) □ U whenever A □ U and U is g- open in (X, τ).
3) *generalized star star closed* (briefly *g**-closed*)**[5]** if cl(A) □ U whenever A □ U and U is g*- open in (X, τ).

The class of g**-open sets of (X,τ) is denoted by G**O(X, τ)

Definition 2.2:[4] An ideal I on a non empty set X is a collection of subsets of X which satisfies the following properties.(i) $A \in I$, $B \in I \Rightarrow A \cup B \in I$ (ii) $A \in I$, $B \subset A \Rightarrow B \in I$ .A topological space (X,τ) with an ideal I on X is called an ideal topological space and is denoted by (X,τ,I).

Definition 2.3:[1] If X is a set and τ_1 and τ_2 are two topologies on X then the triple (X, τ_1, τ_2) is defined to be a bitopological space.

Definition 2.5:[2] A cover U of the topological space (X,τ_1,τ_2) is said to be pairwise open if U $\subseteq \tau_1 \cup \tau_2$ and U contains a non empty member of τ_1 and a non empty member of τ_2.

Definition 2.6:[2] If each pairwise open cover has a finite sub cover then the space is said to be pairwise compact.

Definition 2.7:[10] If each pairwise open cover has a countable sub cover then the space is said to be pairwise lindeloff.

Definition 2.9:[7] A topological space (X,τ) is said to be $g**-multiplicative$ if arbitrary intersection of $g**-closed$ sets is $g**-closed$. Equivalently arbitrary union of $g**-open$ sets is $g**-open$.

Definition 2.11:[8] An ideal topological space (X,τ,I) is said to be g**-compact modulo I if for every g**-open covering $\{U_\alpha\}_{\alpha\in\Delta}$ of X, there exists a finite subset Δ_0 of Δ such that $X - \bigcup_{\alpha\in\Delta_0} U_\alpha \in I$.

Definition 2.12:[8] An ideal topological (X,τ,I) is said to be g**-countably compact modulo I if for every countable g**-open covering $\{U_\alpha\}_{\alpha\in\Delta}$ of X, there exists a finite subset Δ_0 of Δ such that $X - \bigcup_{\alpha\in\Delta_0} U_\alpha \in I$.

Definition 2.13: [9] An ideal topological space (X,τ) is said to be g**-Lindelof modulo I if for every g**-open cover $\{U_\alpha\}_{\alpha\in\Omega}$, there exists a countable subset Ω_0 such that $X - \bigcup_{\alpha\in\Omega_0} \in I$.

Definition 2.14:[2] An ideal topological (X,τ,I) is said to be compact modulo I if for every open covering $\{U_\alpha\}_{\alpha\in\Delta}$ of X, there exists a finite sub collection $\{U_{\alpha_i} / i = 1,2.......n\}$ such that $X - \bigcup_{i=1}^{n} U_{\alpha_i} \in I$.

Definition 2.15:[10] Let (X,τ_1,τ_2) be a bitopological space and I be an ideal in X. Then (X,τ_1, τ_2, I) is said to be pairwise Lindeloff modulo I if every pairwise open cover of X admits a countable sub cover.

Definition 2.16:[9] Let I be an ideal on X and A $\subseteq \wp(X)$. Then A is said to have finite intersection property modulo I (denoted as I – FIP) if for every finite sub family $\{A_i\}_{i=1}^{n}$ of A, $\bigcap_{i=1}^{n} A_i \notin I$.

Definition 2.17:[9] Let I be an ideal on X and A $\subseteq \wp(X)$. Then A is said to have countable intersection property modulo I (denoted as I – CIP) if for every countable sub family $\{A_i\}_{i=1}^{\infty}$ of A, $\bigcap_{i=1}^{\infty} A_i \notin I$.

3. Pairwise g**-comapct modulo *I*, pairwise g**-countably compact modulo *I*, pairwise g**-Lindeloff modulo *I*

Definition 3.1: An ideal bitopological space (X,τ_1,τ_2,I) is called pairwise g**-Lindeloff modulo I if every pairwise g**-open cover $\Im = \{U_\alpha / \alpha \in \Omega\}$ of X has a countable sub collection $\{U_{\alpha_i} / i = 1,2,3...............\}$ of $\Im$ such that $X - \bigcup_{i=1}^{\infty} U_{\alpha_i} \in I$.

Definition 3.2: An ideal bitopological space (X,τ_1,τ_2,I) is called pairwise g**-compact modulo I if every pairwise g**-open cover $\Im = \{U_\alpha / \alpha \in \Omega\}$ of X has a finite sub collection $\{U_{\alpha_i} / i = 1,2,3.............n\}$ of $\Im$ such that $X - \bigcup_{i=1}^{n} U_{\alpha_i} \in I$.

Definition 3.3: A bitopological space (X,τ_1,τ_2) is called pairwise g**-countably compact if every countable pairwise g**-open cover $\Im = \{U_\alpha / \alpha \in \Omega\}$ of X has a finite sub collection $U_{\alpha_i}, i = 1,2,3.............n$ of $\Im$ such that $X - \bigcup_{i=1}^{n} U_{\alpha_i} \in I$.

Theorem 3.4: Every pairwise g**-compact modulo space I is pairwise g**-countably compact modulo I.

Proof is obvious.

Theorem 3.5: Every pairwise g**-compact modulo I space is pairwise g**-Lindeloff Proof is obvious.

Theorem 3.6: Every pairwise g**-Lindeloff modulo I space is pairwise Lindeloff modulo I.

Proof is obvious.

Theorem 3.7: Every pairwise g**-compact modulo I space is pairwise compact modulo I.

Proof is obvious.

Theorem 3.8: Every pairwise g**-countably compact modulo I space is pairwise countably compact modulo I.

Proof is obvious.

Theorem 3.9: Every pairwise g**-Lindeloff modulo I , pairwise g**-countably compact modulo I space is pairwise g**-compact modulo I .

Proof is obvious.

Definition 3.10: Let V be a subset of the bitopological space (X,τ_1,τ_2). Define $\tau_i(V)=\{\varphi, X, U\cup V / U\in\tau_i\}$.

Theorem 3.11: Let (X,τ_1,τ_2,I) be a bitopological ideal space. (X,τ_1,I) and (X,τ_2,I) be g**-multiplicative. Then the following statements are equivalent:

1. (X,τ_1,τ_2) is pairwise g**-compact modulo I.
2. For each non empty $G\in G^{**}O(X,\tau_1,I)$ and $H\in G^{**}O(X,\tau_2,I)$, $(X,\tau_1(H,I))$ and $(X,\tau_2(G),I)$ are g**-compact modulo I.
3. Each g**-closed proper subset of (X,τ_1,I) is g**-compact modulo I in (X,τ_2,I) and each g**-closed proper subset of X is g**-compact modulo I in (X,τ_1,I).

Proof: $1\Rightarrow 2$: Let $G\in G^{**}O(X,\tau_1,I)$ and let A be a open cover for $(X,\tau_2(G),I)$. Then A $=\{G\cup U_\alpha / U_\alpha\in G^{**}O(X,\tau_2,I)\}$. Then $G\cup\{U_\alpha\}_{\alpha\in\Omega}$ is a pirwise g**-open cover for X. Then there exists a finite sub cover $G\cup\{\bigcup_{i=1}^{n} U_{\alpha_i}\}$ such that $X-\{G\cup\{\bigcup_{i=1}^{n} U_{\alpha_i}\}\}\in I$. Therefore $(X,\tau_2(G),I)$ is compact modulo I. Similarly $(X,\tau_1(H),I)$ is compact modulo I.

$2\Rightarrow 3$: Let K be a proper g**-closed set in (X,τ_1,I) then $G=X-K$ is g**-open in (X,τ_1,I). Let $\{V_\alpha\}$ be a g**-open cover for (X,τ_2,I) then $\{G\cup\{V_\alpha\}\}$ is a open cover for $(X,\tau_2(G),I)$. Then there exists $V_{\alpha_1}, V_{\alpha_2},..............V_{\alpha_n}$ such that $X-\{G\cup\bigcup_{i=1}^{n} V_{\alpha_i}\}\in I\Rightarrow (X-G)-\bigcup_{i=1}^{n} V_{\alpha_i}\in I$.. That is $K-\bigcup_{i=1}^{n} V_{\alpha_i}\in I$ Therefore K is g**-compact modulo I in (X,τ_2,I).

$3\Rightarrow 1$: Let A be a pairwise g**-open cover for X. Let τ_1-g**-open set in A be $\{U_\beta / \beta\in\Omega_1\}$ and τ_2-g**-open set in A be $\{V_\alpha / \alpha\in\Omega_2\}$.

Case 1: Let $\bigcup_{\alpha\in\Omega_2} V_\alpha = X$. Choose $\beta_0\in\Omega_1$ such that $U_{\beta_0}\neq\varphi$. Then

$\{V_\alpha / \alpha \in \Omega_2\}$ is a τ_2-g**-open cover for τ_1-g**-closed set $X - U_{\beta_0}$. By (3), there exists

$\therefore X - \{\overset{n}{\underset{i=1}{\cup}} V_{\alpha_i} \cup U_{\beta_0}\} \in I$. Hence (X,τ_1,τ_2,I) is pairwise g**-compact modulo *I*.

Case 2: Let $\underset{\alpha\in\Omega_2}{\cup} V_\alpha \neq X$. Therefore $K = X - \underset{\alpha\in\Omega_2}{\cup} V_\alpha$ is a τ_2-g**-closed set (since (X,τ_2,I) is g**-multiplicative) in X and $K \subseteq X - \underset{\beta\in\Omega_1}{\cup} U_\beta$. By (3), there exists $U_{\beta_1}, U_{\beta_2}, \ldots\ldots\ldots\ldots U_{\beta_n}$ such that $K - \overset{n}{\underset{i=1}{\cup}} U_{\beta_i} \in I$. Suppose $X - \overset{n}{\underset{i=1}{\cup}} U_{\beta_i} \in I$ then (X,τ_1,τ_2,I) is pairwise g**-compact modulo *I*. Suppose $X - \overset{n}{\underset{i=1}{\cup}} U_{\beta_i} \notin I$ then $X - \overset{n}{\underset{i=1}{\cup}} U_{\beta_i} \neq \varphi$ and hence $X - \overset{n}{\underset{i=1}{\cup}} U_{\beta_i}$ is a proper $\tau_1 -$g**closed set of X contained in $\underset{\alpha\in\Omega_2}{\cup} V_\alpha$. By (3), there exists $V_{\alpha_{1,}} V_{\alpha_{12,}} \ldots\ldots\ldots V_{\alpha_k}$ such that $(X - \overset{n}{\underset{i=1}{\cup}} U_{\beta_i}) - (\overset{k}{\underset{j=1}{\cup}} V_{\alpha_j}) \in I$ Then $X - \{(\overset{n}{\underset{i=1}{\cup}} U_{\beta_i}) \cup (\overset{k}{\underset{j=1}{\cup}} V_{\alpha_j})\} \in I$ therefore (X,τ_1,τ_2,I) is pairwise g**-compact modulo *I*.

Theorem 3.12: Let (X,τ_1,τ_2,I) be a bitopological ideal space. (X,τ_1,I) and (X,τ_2,I) be g**-multiplicative. Then the following statements are equivalent:

1. (X,τ_1,τ_2,I) is pairwise g**-Lindeloff modulo *I*.

$V_{\alpha_1}, V_{\alpha_2}, \ldots\ldots\ldots\ldots V_{\alpha_n}$ such that $(X - U_{\beta_0}) - \overset{n}{\underset{i=1}{\cup}} V_{\alpha_i} \in I$.

2. For each non empty $G \in G^{**}O(X,\tau_1,I)$ and $H \in G^{**}O(X,\tau_2,I)$, $(X,\tau_1(H),I)$ and $(X,\tau_2(G),I)$ are Lindeloff modulo *I*.
3. Each g**-closed proper subset of (X,τ_1,I) is g**- Lindeloff modulo *I* in (X,τ_2,I) and each g**-closed proper subset of is g**- Lindeloff modulo *I* in (X,τ_1).

Proof: Similar to the proof of theorem (4.11).

Theorem 3.13: Let (X,τ_1,τ_2,I) be a bitopological ideal space. (X,τ_1,I) and (X,τ_2,I) be finitely g**-additive and g**-multiplicative. Then the following statements are equivalent:

1. (X,τ_1,τ_2,I) is pairwise g**-countably compact modulo *I*.
2. For each non empty $G \in G^{**}O(X,\tau_1,I)$ and $H \in G^{**}O(X,\tau_2,I)$, $(X,\tau_1(H),I)$ and $(X,\tau_2(G),I)$ are countably compact modulo *I*.
3. Each g**-closed proper subset of (X,τ_1,I) is g**- countably compact modulo *I*. in (X,τ_2,I) and each g**-closed proper subset of (X,τ_2,I) is g**- countably compact modulo *I* in (X,τ_1,I).

Proof: Similar to the proof of theorem (4.11).

Theorem 3.14: Let (X,τ_1,τ_2,I) be a bitopological ideal space, then the following statements are equivalent:

1. (X,τ_1,τ_2,I) is pairwise g**-compact modulo *I*.
2. For every family $\{F_\alpha/\alpha\in\Omega\}$ of pairwise g**-closed sets such that $\bigcap_{\alpha\in\Omega}F_\alpha=\varphi$, there exists a finite sub family $\{F_{\alpha_i}/i=1,2,..........n\}$ such that $\bigcap_{i=1}^{n}F_{\alpha_i}\in I$.
3. For every family $\{F_\alpha/\alpha\in\Omega\}$ of pairwise g**-closed sets with *I*-FIP, $\bigcap_{\alpha\in\Omega}F_\alpha\neq\varphi.$

Proof: $1\Rightarrow 2$:Let $\{F_\alpha/\alpha\in\Omega\}$ be a family of pairwise g**-closed sets such that $\bigcap_{\alpha\in\Omega}F_\alpha=\varphi$. Then the collection $\{X-F_\alpha/\alpha\in\Omega\}$ is a pairwise g**-open cover of X. by (1), there exists a finite sub collection $\{X-F_{\alpha_i}/i=1,2.....n\}$ such that $X-\bigcup_{i=1}^{n}(X-F_{\alpha_i})\in I$. Therefore $\bigcap_{i=1}^{n}F_{\alpha_i}\in I$.

$2\Rightarrow 3$:Let Let $\{F_\alpha/\alpha\in\Omega\}$ be a family of pairwise g**-closed setswith *I*-FIP. If $\bigcap_{\alpha\in\Omega}F_\alpha=\varphi$. By (2), there exists a finite sub family $\{F_{\alpha_i}/i=1,2,..........n\}$ such that $\bigcap_{i=1}^{n}F_\alpha\in I$ which is a contradiction. Therefore $\bigcap_{\alpha\in\Omega}F_\alpha\neq\varphi.$

$3\Rightarrow 1$:Let U $=U_\alpha/\alpha\in\Omega\}$ be a pairwise g**-open cover of X. To prove U has a finite sub collection $\{U_{\alpha_i}/i=1,2........n\}$ such that $X-\bigcup_{i=1}^{n}U_{\alpha_i}\in I$.If not, then $\{X-U_\alpha/\alpha\in\Omega\}$ is a family of pairwise g**-closed sets such that $\bigcap_{i=1}^{n}\{X-U_\alpha\}\notin I$ for any finite subset of Ω . By (3), $\bigcap_{\alpha\in\Omega}\{X-U_\alpha\}\neq\varphi.$ $\therefore X\neq\bigcup_{\alpha\in\Omega}U_\alpha$ which is a contradiction. Therefore U has a finite sub cover modulo *I*. Hence (X,τ_1,τ_2) is pairwise g**-compact modulo *I*.

Theorem 3.15: Let (X,τ_1,τ_2,I) be a bitopological ideal space. Then the following statements are equivalent:

1. (X,τ_1,τ_2,I) is pairwise g**-Lindeloff modulo *I*.
2. For every family $\{F_\alpha/\alpha\in\Omega\}$ of pairwise g**-closed sets such that $\bigcap_{\alpha\in\Omega}F_\alpha=\varphi$, there exists a countable sub family $\{F_{\alpha_i}/i=1,2,..........\}$ such that $\bigcap_{i=1}^{\infty}F_{\alpha_i}\in I$.
3. For every family $\{F_\alpha/\alpha\in\Omega\}$ of pairwise g**-closed sets with *I*-CIP, $\bigcap_{\alpha\in\Omega}F_\alpha\neq\varphi.$

Proof: Similar to the proof of theorem (4.14).

Theorem 3.16: Let (X,τ_1,τ_2,I) be a bitopological ideal space. Then the following statements are equivalent:

1. (X,τ_1,τ_2,I) is pairwise g**-countably compact modulo *I*.
2. For every family $\{F_\alpha/\alpha\in\Omega\}$ of pairwise g**-closed sets such that $\bigcap_{\alpha\in\Omega}F_\alpha=\varphi$, there exists a countable

sub family $\{F_{\alpha_i} / i = 1,2,.........\}$ such that $\bigcap_{i=1}^{\infty} F_{\alpha_i} \in I$.

3. For every family $\{F_\alpha / \alpha \in \Omega\}$ of pairwise g**-closed sets with *I*-CIP, $\bigcap_{\alpha\in\Omega} F_\alpha \neq \varphi$.

Proof: Similar to the proof of theorem(4.14).

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